Hong Kong Mathematics Olympiad (1999 – 2000)
Final Event 1 (Group)
香港數學競賽 (1999 – 2000)
決賽項目 1 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 已知整數 n 除 81849、106392 及 124374 得出的餘數相等，求 n 的最大值 a。

Given that when 81849, 106392 and 124374 are divided by an integer n, the remainders are equal. If a is the maximum value of n, find a.

$a =$

(ii) 設 $x=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ 及 $y=\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$。如果 $b=2x^2-3xy+2y^2$，求 b 的值。

Let $x=\dfrac{1-\sqrt{3}}{1+\sqrt{3}}$ and $y=\dfrac{1+\sqrt{3}}{1-\sqrt{3}}$. If $b=2x^2-3xy+2y^2$, find the value of b.

$b =$

(iii) 已知 c 為正數，如果只有一條直線穿過點 $A(1, c)$ 且與曲線 C：$x^2+y^2-2x-2y-7=0$ 相交於一點，求 c 的值。

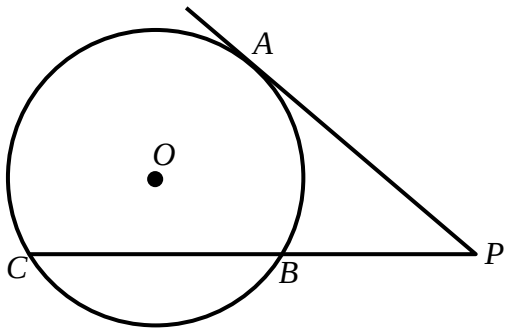
Given that c is a positive number. If there is only one straight line which passes through point $A(1, c)$ and meets the curve C: $x^2+y^2-2x-2y-7=0$ at only one point, find the value of c.

$c =$

(iv) 在圖一，PA 切圓於 A，O 為圓心。如果 $PA=6$，$BC=9$，$PB=d$，求 d 的值。

$d=$

In Figure 1, PA touches the circle with center O at A. If $PA=6$, $BC=9$, $PB=d$, find the value of d.



圖一

Figure 1

Hong Kong Mathematics Olympiad (1999 – 2000)
Final Event 2 (Group)
香港數學競賽 (1999 – 2000)
決賽項目 2 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 191 為兩個連續平方數之差，而 a 為其中最小的平方數，求 a 的值。

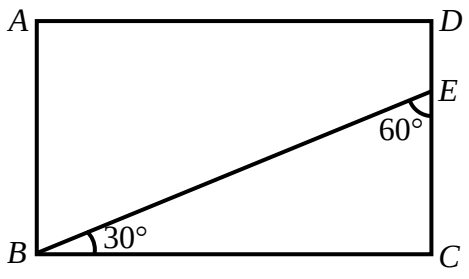
If 191 is the difference of two consecutive perfect squares , find the value of the smallest square number, a .

$a =$

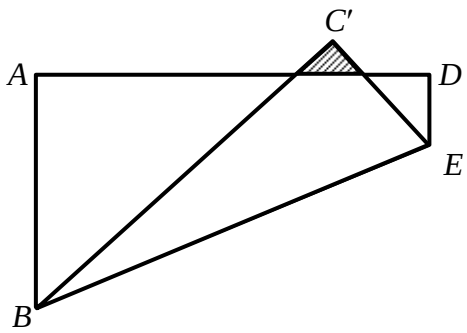
(ii) 在圖二(a)，$ABCD$ 是一長方形。$DE : EC = 1 : 5$，且 $DE = 12^{\frac{1}{4}}$。ΔBCE 沿 BE 摺去另一方。設 b 為圖二(b)中陰影部份的面積，求 b 的值。

In Figure 2(a), $ABCD$ is a rectangle. $DE : EC = 1 : 5$, and $DE = 12^{\frac{1}{4}}$. ΔBCE is folded along the side BE . If b is the area of the shaded part as shown in Figure 2(b), find the value of b .

$b =$



圖二(a)
Figure 2(a)



圖二(b)
Figure 2(b)

(iii) 設曲線 $y = x^2 - 7x + 12$ 與 x 軸的交點為 A 及 B，而與 y 軸的交點為 C。如果 c 是ΔABC 的面積，求 c 的值。

Let the curve $y = x^2 - 7x + 12$ intersect the x-axis at points A and B, and intersect the y-axis at C. If c is the area of ΔABC, find the value of c.

$c =$

(iv) 設 $f(x) = 41x^2 - 4x + 4$，$g(x) = -2x^2 + x$。如果 $f(x) + kg(x) = 0$ 只有一個根，求 k 的最小值 d。

Let $f(x) = 41x^2 - 4x + 4$ and $g(x) = -2x^2 + x$. If d is the smallest value of k such that $f(x) + kg(x) = 0$ has a single root , find d.

$d =$

Hong Kong Mathematics Olympiad (1999 – 2000)
Final Event 3 (Group)
香港數學競賽 (1999 – 2000)
決賽項目 3 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

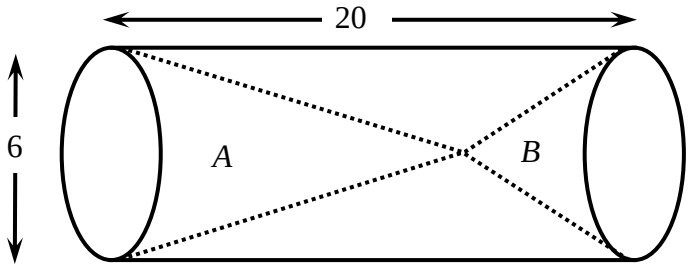
(i) 設 $a=\sqrt{1997\times1998\times1999\times2000+1}$，求 a 的值。

Let $a=\sqrt{1997\times1998\times1999\times2000+1}$, find the value of a.

$a=$

(ii) 在圖三，圓管的長為 20 及直徑為 6，內有兩個圓錐體 A 和 B。A 及 B 的體積比例為 $3:1$。如果 b 是 B 的高度，求 b 的值。

In Figure 3, A and B are two cones inside a cylindrical tube with length of 20 and diameter of 6. If the volumes of A and B are in the ratio $3:1$ and b is the height of the cone B, find the value of b.



圖三
Figure 3

$b=$

(iii) 現有點 $A\left(\frac{\sqrt{10}}{2},\frac{\sqrt{10}}{2}\right)$ 和圓 $C: x^2+y^2=1$。如果 c 是通過點 A 與圓相切直線的最大斜率，求 c 的值。

If c is the largest slope of the tangents from the point $A\left(\frac{\sqrt{10}}{2},\frac{\sqrt{10}}{2}\right)$ to the circle $C: x^2+y^2=1$, find the value of c.

$c=$

(iv) 在坐標平面的原點上有一點 P。假如擲出骰子的點數 n 是偶數，P 在 x 方向右前進 n；如果 n 是奇數，P 在 y 方向上前進 n。如果有 d 種不同擲法使得 P 到達點 $(4, 4)$，求 d 的值。

P is a point located at the origin of the coordinate plane . When a dice is thrown and the number n shown is even , P moves to the right by n . If n is odd, P moves upward by n . Find the value of d , the total number of tossing sequences for P to move to the point $(4, 4)$.

$d =$

Hong Kong Mathematics Olympiad (1999 – 2000)
Final Event 4 (Group)
香港數學競賽 (1999 – 2000)
決賽項目 4 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 a 是一個三位數，駁在 504 之後，新組成的六位數可被 7 、 9 、 11 整除，求 a 的值。

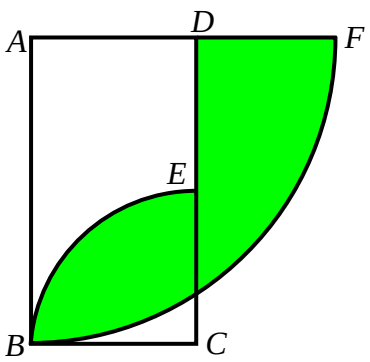
Let a be a 3-digit number . If the 6-digit number formed by putting a at the end of the number 504 is divisible by 7 , 9 , and 11 , find the value of a .

$a =$

(ii) 在圖四，$ABCD$ 為長方形，$AB = \sqrt{\dfrac{8+\sqrt{64-\pi^2}}{\pi}}$ ，$BC = \sqrt{\dfrac{8-\sqrt{64-\pi^2}}{\pi}}$ 。BE、BF 分別是以 C 、 A 為圓心的弧。若 b 是陰影部份之面積，求 b 的值。

In Figure 4 , $ABCD$ is a rectangle with $AB = \sqrt{\dfrac{8+\sqrt{64-\pi^2}}{\pi}}$ and $BC = \sqrt{\dfrac{8-\sqrt{64-\pi^2}}{\pi}}$. BE and BF are the arcs of circles with centers at C and A respectively . If b is the total area of the shaded parts, find the value of b .

$b =$

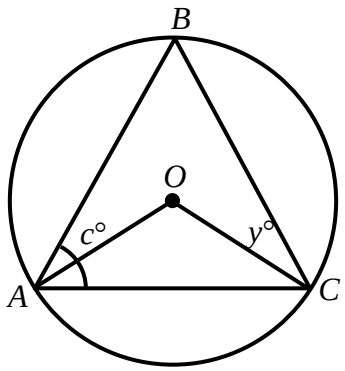


圖四
Figure 4

(iii) 在圖五，O 為圓心，$c° = 2y°$，求 c 的值。

In Figure 5, O is the centre of the circle and $c° = 2y°$. Find the value of c.

$c =$



圖五

Figure 5

(iv) A、B、C、D、E、F、G 七個人圍圓桌而坐。如果 B 及 G 都與 C 相鄰而坐的坐法總數為 d，求 d 的值。

A, B, C, D, E, F, G are seven people sitting around a circular table. If d is the total number of ways that B and G must sit next to C, find the value of d.

$d =$

Hong Kong Mathematics Olympiad (1999 – 2000)
Final Event 5 (Group)
香港數學競賽 (1999 – 2000)
決賽項目 5 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

(i) 如果 a 是可被 810 整除的最小立方數，求 a 的值。

If a is the smallest cubic number divisible by 810 , find the value of a .

$a =$

(ii) 設 b 是函數 $y = |x^2 - 4| - 6x$ (其中 $-2 \le x \le 5$) 的最大值，求 b 的值。

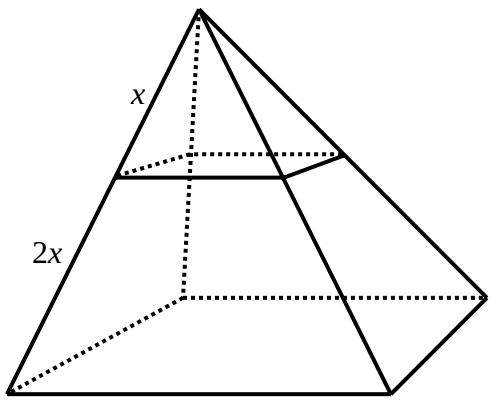
Let b be the maximum of the function $y = |x^2 - 4| - 6x$ (where $-2 \le x \le 5$) , find the value of b .

$b =$

(iii) 圖六為一個正方形底的錐體。若從底部向上並在 $\frac{2}{3}$ 之高度平行橫切，並設 $1 : c$ 為上面細錐與餘下底部體積的比，求 c 的值。

In Figure 6 , a square-based pyramid is cut into two shapes by a cut running parallel to the base and made $\frac{2}{3}$ of the way up . Let $1 : c$ be the ratio of the volume of the small pyramid to that of the truncated base, find the value of c .

$c =$



圖六
Figure 6

(iv) 如果 $\cos^6\theta+\sin^6\theta=0.4$，及 $d=2+5\cos^2\theta\sin^2\theta$，求 d 的值。

If $\cos^6\theta+\sin^6\theta=0.4$ and $d=2+5\cos^2\theta\sin^2\theta$, find the value of d.

$d=$